Perturbation Method for Low-Frequency Calderón Multiplicative Preconditioned EFIE

Sheng Sun ¹, Weng Cho Chew ¹², Yang G. Liu ¹³, Zuhui Ma ¹

¹ Department of Electrical and Electronic Engineering
The University of Hong Kong, Pokfulam, Hong Kong, China
sunsheng@hku.hk

² Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA
w-chew@uiuc.edu

³ Institute of Applied Physics and Computational Mathematics
No.2 Fenghao East Road, Beijing 100094, China

Abstract: This paper addresses the low-frequency problems in the electric field integral equation (EFIE) with the Calderón multiplicative preconditioner (CMP). Based on the self-regularizing property of the EFIE and Calderón identities, CMP-EFIE yields a very well-conditioned system matrix and is stable at low frequencies. However, the roundoff error of CMP-EFIE cannot be avoided when the frequency goes to zero. To avoid using the loop-star decomposition, we propose a perturbation method as a remedy to remove the error and improve the accuracy at low frequencies. Numerical examples demonstrate the low-frequency electric current at different orders can be correctly captured, thus leading to an accurate far-field computation.

Keywords: Calderón multiplicative preconditioner (CMP), electric field integral equation (EFIE), electromagnetic scattering, low frequency, perturbation method.

1. Introduction

Electric field integral equation (EFIE) has been extensively used in analyzing the electromagnetic (EM) radiation and scattering problems for a perfect electric conductor (PEC) object. However, the numerical solution of the EFIE at low frequencies suffers from the breakdown and inaccuracy problems. The low-frequency breakdown problem happens when the Rao-Wilton-Glisson (RWG) basis function is employed in EFIE. Physically, it is because of the decoupling between the electric field and magnetic field [1]. In the integral representation of the EFIE, the electric field is decomposed into the vector potential part and the scalar potential part. At low frequencies, the vector potential part is much smaller than the scalar potential part. Hence, the contribution of the vector potential part will be lost during the numerical process. Furthermore, the remaining scalar potential part has a null space, which makes the system matrix extremely ill-conditioned [2]. On the other hand, the loss of the contribution from the vector potential leads to an inaccurate solution [3], [4]. In order to overcome these problems, the loop-star/-tree basis function [3]-[9] was proposed to separate the contributions from the vector potential and the scalar potential in the EFIE formulation. However, it requires loop search, which is especially...
difficult for a complex structure with many entangled long loops. In recent years, many methods have been introduced to reduce the condition number and improve the convergence of EFIE at low frequencies, such as the augmented EFIE (A-EFIE) [10] and the Calderón multiplicative preconditioner (CMP) [11]. By including charge as additional unknown in the A-EFIE, a simple remedy has been proposed to avoid the imbalance between vector potential and the scalar potential, thus solving the low-frequency breakdown problem [10]. However, the A-EFIE loses accuracy of current at low frequencies for certain applications such as plane wave scattering. To address the low-frequency inaccuracy problem, a perturbation method has been further proposed to obtain the current and charge on different frequency orders as a perturbation series [12]. Alternatively, applying the CMP [11] on the EFIE yields a well-conditioned system matrix and the resultant operator is stable at low frequencies. However, this method still suffers from the low-frequency problems [13]. In this paper, we propose the perturbation method as a remedy for the CMP-EFIE at low frequencies. Without doing the loop-star decomposition [14], [15], the CMP-EFIE with perturbation method can work at very low frequencies accurately. The calculated electric currents at different orders also show a good agreement with those obtained from the A-EFIE with perturbation [12].

2. Background

The traditional EFIE operator in the integral representation can be written in its mixed potential form as

\[
\mathcal{T}(\mathbf{J}) = i\omega \mu \mathbf{n} \times \iint_S g(r,r') \mathbf{J} dS' - {1 \over i\omega \epsilon} \mathbf{n} \times \nabla \iint_S g(r,r') \nabla' \cdot \mathbf{J} dS'
\]

\[= \mathcal{T}_s(\mathbf{J}) + \mathcal{T}_h(\mathbf{J}) \tag{1}\]

where \(g\) denotes the free-space Green’s function, \(\epsilon\) and \(\mu\) are the relative permeability and permittivity, \(\mathbf{J}\) is the unknown surface current on the scatterer, and the subscript \(s\) and \(h\) indicate the smoothing and hypersingular terms, respectively, which are also phrased as the vector potential part and scalar potential part. When the frequency approaches zero, the smoothing term goes to zero and the hypersingular term goes to infinity. Because of a null space of the divergence operator in the above hypersingular term, the system matrix becomes singular and extremely ill-conditioned at low frequencies. Meanwhile, the contribution from the smoothing term will be swamped by that from the hypersingular term at low frequencies, which makes the solution inaccurate [2]-[4]. In recent years, different preconditioners have been introduced to improve the spectral property of the EFIE operator. Among them, the CMP based on the self-regularizing property and Calderón identities is one of the most effective ways to reduce the condition number of the system matrix. The Calderón identity used for the EFIE is

\[
\mathcal{T}^2(\mathbf{J}) = -{\mathbf{J} \over 4} + \mathcal{K}_s(\mathbf{J}) \tag{2}\]

where

\[
\mathcal{K}_s(\mathbf{J}) = \mathbf{n} \times \nabla \times \iint_S g(r,r') \mathbf{J} dS' \tag{3}\]

is the magnetic field integral equation (MFIE) operator [1]. Obviously, the composite operator \(\mathcal{T}^2\) is a second-kind integral operator and has a bounded spectrum because of the property \(\mathcal{T}^2_h = 0\). When the frequency is not very low, the \(\mathcal{T}^2_h = 0\) property can be well conserved even after discretization of \(\mathcal{T}^2\). However, the numerical cancellation errors accumulate rapidly when the frequency approaches zero [13]. One way to remedy this issue is to do the loop-star decomposition [14], [15], but the Gram matrix is no longer well-conditioned due to the overlap of the large domain of support of loop and star basis functions. Another way is to decompose \(\mathcal{T}^2\) into four terms and to omit the contribution from the \(\mathcal{T}^2_h\) term.
manually as
\[ T^2 = T_s^2 + T_h T_s + T_h^2 \]  \hspace{1cm} (4)

Although this decomposition scheme involves more matrix vector products, it is unconditionally stable at very low frequencies [13]. Unfortunately, we observed that it cannot guarantee the accuracy of the results when the frequency becomes extremely low.

3. Perturbation Method

In order to enhance the accuracy of the aforementioned CMP-EFIE, we propose a perturbation method, which has been successfully applied in the A-EFIE [12] for low-frequency electromagnetic scattering problems. The idea of the perturbation method is to obtain approximate solutions to problems involving a small parameter. For the EFIE operator, the small parameter \( \delta = i k_0 \) can be defined in the expansion of the Green’s function:
\[ g(r,r') = \frac{1}{4\pi R} \left[ 1 + i k_0 R + \frac{1}{2} (i k_0 R)^2 \right] + O(\delta^3) \]  \hspace{1cm} (5)

Then, the sub-matrices, the current and excitation vectors after the discretization procedure can be expanded with respect to the small parameter \( \delta = i k_0 \):
\[ \overline{Z} = \overline{Z}^{(0)} + \delta \overline{Z}^{(1)} + \delta^2 \overline{Z}^{(2)} + O(\delta^3) \]  \hspace{1cm} (6)
\[ i k_0 j = j^{(0)} + \delta j^{(1)} + \delta^2 j^{(2)} + O(\delta^3) \]  \hspace{1cm} (7)
\[ \mathbf{b} = \mathbf{b}^{(0)} + \delta \mathbf{b}^{(1)} + \delta^2 \mathbf{b}^{(2)} + O(\delta^3) \]  \hspace{1cm} (8)

Matching the coefficients of like powers of \( \delta \), we obtain a recurrent system of equations for the electric current functions \( j \). Subsequently, the low-frequency current at different orders can be accurately captured, thus improving the accuracy of the far-field results.

4. Numerical Results

We solve the CMP-EFIE for EM scattering by a PEC sphere based on the same discretization procedure using the Buffa-Christiansen (BC) basis function [11], which is a linear combination of RWG basis function and approximately orthogonal to the original div-conforming RWG basis function. We notice that the BC basis function represents a subset of the functions proposed by Chen and Wilton in 1990 [16], which were named the dual basis. Since the idea of these two basis functions is completely the same, we should rightfully call them the Chen-Wilton-Buffa-Christiansen (CWBC) basis function. In the first example, an x-polarized plane wave impinges onto a PEC sphere from the +z direction. The sphere centers at the origin and has a radius of 1 m. We discretize the surface into 578 triangular patches, equivalent to 867 inner edges. At 1 Hz, the initial CMP-EFIE system in [11] breaks down, and the performance obtained from the decomposed CMP-EFIE in (4) is wrong due to the inaccuracy of low-frequency current. As shown in Fig. 1(a), the decomposed CMP-EFIE with perturbation is still able to deliver the right results as those obtained by Mie series analytically. Next, we did a comparison for the currents at different orders obtained from the CMP-EFIE with perturbation method and the A-EFIE in [12]. As shown in Fig. 1(b), a good agreement has been obtained between them.

In the second example, we replace the sphere with a PEC torus shown in Fig. 2(a), which has two global loops. The radius of the tube is 0.1 m and the distance from the center of the tube to the center of the torus is 0.3 m. We discretize the surface into 1,066 triangular patches, equivalent to 1,599 inner edges. Fig. 2(b) shows the comparison of RCS between A-EFIE [12] and CMP-EFIE with perturbation at 1 Hz. Similar to the sphere case, CMP EFIE with perturbation generates correct results as the A-EFIE with perturbation does.
7. Conclusions

In this paper, we have addressed the low-frequency problems in the CMP-EFIE and enhanced the accuracy of the electric current through the perturbation method. Different from the previous solution with the loop-star decomposition [14], [15], the proposed method keeps the original sparse characteristic of the Gram matrix in the CMP-EFIE and avoid loop search. Numerical examples have shown that the far-field results can be computed accurately at very low frequencies.

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References


